# NEWTON THEREAFTER

## Viewpoint on Fractal and Complexity

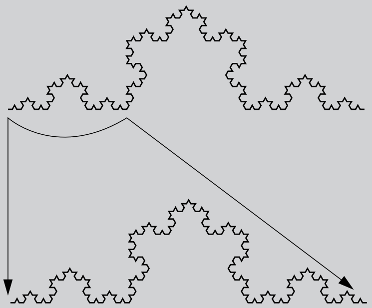
***The starting-out points are the simple questions a child asks his parents.***
***"Why do clouds assume the shapes they do?"***
***"Why do lightnings run zigzag?"***
***In time, these children depart from these doubts.***
***But physics, subsequent to its period of growth, has conversely come to direct its eyes on nearby curiosities.***
***Not content with the orderly world view presented by Newton, it conceives to understand complicated phenomena in their existing states.***
***One of these viewpoints is "fractal."***
***Complexity....chaos....complex systems....they appear understood and yet remain dubious. Persons with such sense of haziness should come out from the fractal domain and step inside the world of complexity.***

### Objection to Classical Physics
### Restoration of Monster

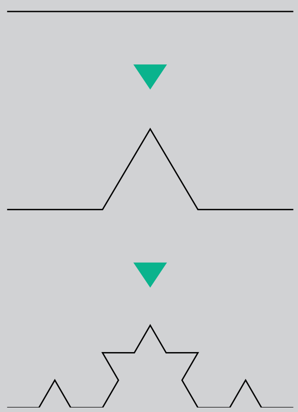
Even with a meandering curve, if a portion of it was expanded on and on, it should ultimately appear like a straight line. Newton established his differential calculus concept on the basis of this perception. Namely, he took up a complicated and unapproachable target, simplified it neatly and manifested it on the stage of research. Newton's thought is the main flow in the field of science, and virtually all theories of physics came to be described with differential equations.
However, about 20 years ago, a researcher called Mundelblow cast a simple shadow of doubt.
"Is it really true that if the curved line was expanded repeatedly, it will always become a straight line?"
For example, let's take the coastline. With a Rias type coast shown jaggedly on a world map, even if we checked it scrupulously with a more detailed map of Japan, the coastline will still appear jagged. Also, if we actually stood on the coastline, rocks would be seen all around, and when the rock surfaces were scrutinized with a microscope, we'll only see jaggedness. If we further expanded the image, we'll have only a group of atoms, and we'll never see a straight line, no matter what we do.
Mundelblow thought that a new concept will be necessary for use in place of the differential calculus concept, and suggested, "What about a diagram that would appear the same no matter how much it was expanded." Searching for such a "self-analogous" diagram, he visited one library after another.
His eyes caught diagrams which mathematicians in the early part of this century called "disease function" or "monster function," which are products of fancy of no practical value. A typical example is Koch's Curve (see page 4, Figure 1).
Koch's curve is drawn in conformance with an extremely simple rule, but even at that, the queer characteristic of this diagram is that whichever part of the diagram is expanded, one got the same original Koch's curve.
In 1975, Mundelblow called all the self-analogous diagrams including the Koch's Curve "fractal" diagrams.
Next, Mundelblow thought of applying the fractal diagram and creating a structure closely resembling those of the natural world. As it is, the Koch's curve was overly orderly, so if it was given some randomness (see Figure 2), then.....
"Wouldn't it appear like a coastline?"
A structure existing in the natural world may be described with the fractal concept, and this fractal concept features the regularity of self-analogy. Namely, behind the veil of non-orderliness, a slight orderliness came to be seen.
Mundelblow also suggested a ruler that would describe complicated structures quantitatively. This is the fractal dimension obtained with a formula. For example, the Koch's curve was of 1.26 dimensions. Namely, it is more complicated than a simple line of one dimension, but not as complicated as to fill up a two-dimensional plane. Taking nature as the example, a tree is generally of 1.5 dimensions, the earth's surface of 2.2 dimensions, the cloud of 2.3 dimensions and the planet distribution of 1.2 dimensions.
The natural world is complicated and difficult to grasp tangibly, but this very complexity enabled the natural world to be described quantitatively, or in the form of "the weight is so many kilograms, the length so many meters and the complexity so many dimensions."

**■ Figure 1　Koch's Curve**

Whichever portion is expanded, the section appears the same as the original (self-analogous). With this Koch's curve, a line of infinite length is folded in lamination inside a finite size. Therefore, a coastline displaying this property will in principle have a length of several times when the measurement unit is shortened.



**How to Draw Koch's Curve**
The given line section is divided equally into three parts, and the middle section replaced with a folded line consisting of two line sections. This operation is repeated on the remaining section sequentially.



## Transcendence of Element Reduction Principle
## Point for Turnback to Micro

After having learned the background of fractal, let's go to meet a front-line researcher. He is Professor Hideki Takayasu who transferred from Tohoku University to Sony Computer Science Laboratory this spring.

His psychology may be sensed from what he said. "My feeling is that, as with an exploration group, to first go forth alone. Taking students along can be done after the direction has been determined." Written down in text form, it may appear like a violent impression, but orally, it will sound quite gentle.

Encouraged by this environment, I came up with a basic question and asked the ways in which the elements of fractal/complex system/chaos are mutually related.

The reply: "Represented in groups, fractal and chaos are overlapped partially, since the variations or structures created by chaos may at times be fractal. And an attempt is being made to grasp and understand the world's complicated phenomena as various forms of chaos complexes, and the attempt may be regarded as research on complex systems."

The underlying thought of complex systems is the often-said "negation of iodometry (element reduction principle)*." I feel I can understand this. Because it is in conformance with the fractal concept of a straight line not being generated, no matter how much the expansion. However, ......

His feeling: "I hold a sense of skepticism on the tendency to say outright that iodometry is unacceptable. This is because I feel that analyzing matters in details is a vital method of approach in science."

Come to think of it, in the fractal concept, element reduction is performed in the same manner by which Koch's curve is reduced. "However, Koch's curve cannot be reduced further from that point, because no matter how much the curve sections are expanded, they are self-analogous. An appropriate expression may be to say that when reduced to a certain extent, the reduction is stopped and a turnback is achieved in the direction of macro."

This is not any negation of classical physics methodology. Without classical physics, the concept of fractal would not have emerged. But conventional physics discovered a state not involving any complexity (state of vacuum, for example), and has extracted the limit characteristics of substances. By contrast, current physics has returned the visual point to the world as it is and trying to understand complexity itself.

As observed from this perspective, the subsequent activities of Newton, who decoded the law of motions of planets, come to assume significance. Newton established the foundation of presentday physics in his early twenties, then faded away from the foreground stage of physics. He confined himself in his laboratory and devoted himself to alchemy experiments. Professor Takayasu says he feels he can understand the secret reason for this action that is regarded as a hidden facet of this exalted scientist.

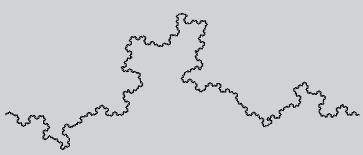
"After having elucidated the motions of planets by differential calculus, Newton set forth to elucidate a world lying nearer to his proximity, but his differential calculus approach was ineffective for clarifying the chaotic world of substances. Therefore, Newton had no alternative but to seek for a way out by burying himself in alchemy that was then the sole sector of science that was handling substances. If he had been born in this age, he may have been leading studies on the science of complexity."

* Iodometry (element reduction principle) The study in which phenomena are resolved into elements, and all explanations offered on the basis of the basic rule between elements. This is based on the tacit world view that complicated matters can be simplified by reduction.

## Laws of Determinism and Probability
## Road for Separation from Chaos

The self-analogy of fractal, which enables the same shape to be seen even if the observation scale was changed, is not limited to spatial structures. For example, changes which occur with a lapse of time as well as the fluctuations of stock market prices, when studied in the form of line graphs and observed in day unit or month unit, will both appear jaggedly and reflect complicated fluctuations. Namely, they are self-

**■ Figure2　Random Koch's Curve**

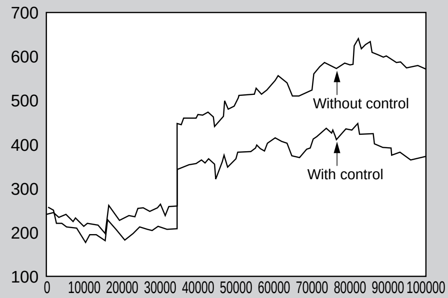


**Hideki Takayasu, Dr. Sci.**
Currently a Senior Researcher at the Sony Computer Science Laboratory, Dr. Takayasu was born in 1958 and received his doctorate in nonlinear physics from Nagoya University. He became a professor in the Information Science department at Tohoku University in 1993 and assumed his present post in April 1997. His publications include "Fractals" and "What are Fractals", the latter being jointly authored with his wife.

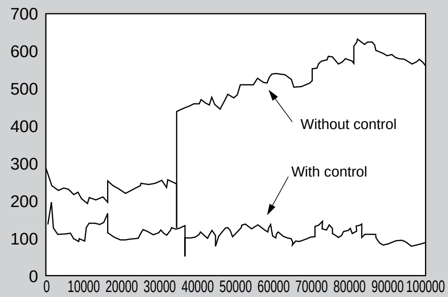
analogous.
Or take, for example, the distribution of sizes. As in the case of stock prices, assuming that the price of a stock changes by over 10 yen for once in ten days, then the probability of a price fluctuation of over 20 yen will be only one-third, or once in thirty days. For a fluctuation of over 40 yen, the probability will be another one-third, or once in 90 days, and a fluctuation of over 80 yen will be once in 270 days. If we proceeded at this ratio, a market crash in which the fluctuation will be over 1,000 yen in a day will inevitably occur once in about 60 years. This is the provision of nature.
If such a distribution was represented by a "fractal distribution" or a numerical formula, the function will assume a power series, hence the name "power distribution." The feeling is that, as observed from the image, the ratio will be the same (one-third in the aforementioned example) no matter which magnitude fluctuation was examined, and that it will be possible to observe that other magnitude fluctuations will be aligned in the same manner. The image will continue infinitely, as when facing mirrors towards each other, and the ultimate conclusion is self-analogy.
The fractal distribution will also apply to the size distribution of other matters such as earthquakes, Milky Way, meteors and companies, also to the frequency distribution of words. When observing these expansions, fractal research will be interdisciplinary, which may be regarded as being the provision of nature.
Here, we come up against a question. If the law of stock fluctuation was known, wouldn't it be possible to forecast future stock prices? Professor Takayasu says, "This is where a difference exists between fractal and chaos."
"The chaos thought is that even a phenomenon appearing only at random may be actually moving in determinism. Therefore, even regarding stock price fluctuations, research is advanced to pursue after fluctuations dynamically from the perspective of determinism and to forecast the future. On the other hand, fractal research is based on probability, so even if an analysis was made on the magnitude of fluctuation and the probability of its degree of occurrence, it will be impossible to determine the specific time of occurrence of such a situation."
Then, would there be no possibility for fractal research to influence the world actively? In this connection, Professor Takayasu is currently advancing a fascinating research project.
"The Bank of Japan frequently intervenes with the aim of controlling the yen's foreign exchange rate, but anticipated results are hardly attained. Therefore, model analysis was conducted and it was learned that the intervention effect will differ widely on the timing of the intervention (see Graph 1). We have assumed up till now that diverse fluctuations are natural occurrences and strove to elucidate the nature of these fluctuations. More recently, it is possible to describe these fluctuations with a simple equation. Based on the knowledge acquired through these attempts, a method is under development that may enable these fluctuations to be controlled effectively."
In the 1990s, fractal research entered its stage of full-scale application. For example, an image compression technology has already been commercialized to decompose complicated images such as scenic photos into several simple fractal components for transmission.
Professor Takayasu has also been engaged in research from several years ago to utilize the fractal dimension for cancer discrimination. The principle is to utilize endoscopic images and to discriminate cancer tissues which are characterized by more complicated structures than normal tissues.
"The fractal concept is fundamental and, depending on the conception, it can be put to a broad range of applications, in the same manner as differential equations."
It appears that the fractal concept cannot be wiped out from Professor Takayasu's mind. His wife is also a researcher in this same field. When Pro-

**■ Graph 1 Analysis of foreign exchange market rate model: When to implement control**

● The fluctuation cannot be controlled by intervention whose timing is not considered.




● Intervention when the fluctuation range is small enables effective control.



fessor Takayasu was asked, "Then, if your friends come to visit your home, do they cannot enter actively into you and your wife's conversation?" To this question, he came out with resounding laughter as usual, and replied, "No, because many of my friends are related to physics."

## Natural Observation, Experiments and Computers Physicist Appears

Professor Takayasu, while a physicist, is engaged in research relating to economic phenomena. In like manner, there is a researcher who is basically a physicist but challenging biological themes. He is Professor Mitsugu Matsushita of the Faculty of Engineering, Chuo University.

He says, "We look at the clouds, for example, or the distant mountain ranges. They are scenes we are accustomed to from our childhood days, but we are unaware that there is a definite regularity in their backgrounds. If we came to know this, wouldn't our field of vision be widened substantially? When the viewpoint of self-analogy was proposed, I was jolted and felt as if my eyes were opened widely."

Professor Matsushita summarizes the footsteps of fractal research as follows.

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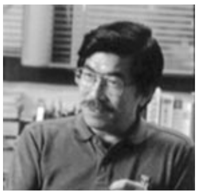
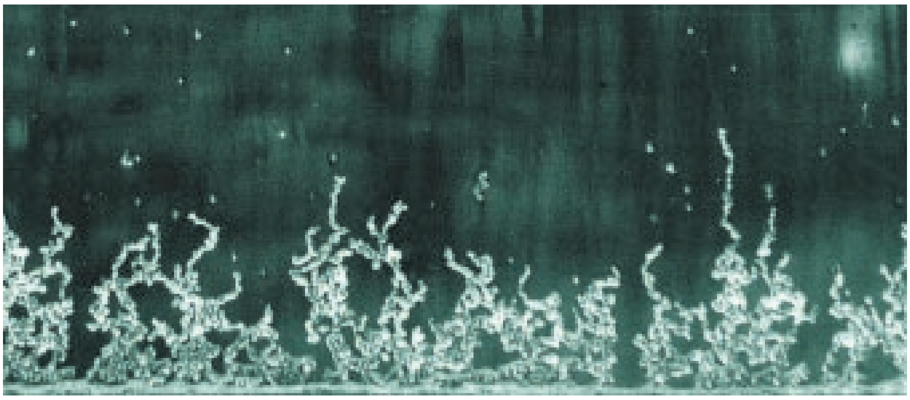
**■ Fractal Prehistory: Up to 1970s**

Particle science in search of quark enters into the stage of experiments with large-sized accelerators. Theoretically, the big van, unified theory and other high-energy matters are discussed which can hardly be handled by actual experiments.



**Mitsugu Matsushita, Dr. Sci.**

Dr. Matsushita is a professor in the Physics Department, Faculty of Science and Engineering, at Chuo University. Born in 1943, Dr. Matsushita completed his doctorate at the University of Tokyo. He has been at his current position since 1987. He is joint author of "Invitation to Numerical Physics, Part 2" and has extensive credits as a translator, including "Introduction to Chaos Dynamics" and "World of Fractals".



▲ The patterns that appear on window glass are also fractals.

Each of these themes is important, but there may have been many scientists who desired to engage in research relating to fields lying closer to themselves.

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**■ Age of Fractal Natural History: Early 1970s**

When Mundelblow proposed the fractal concept in 1975, researchers belonging to the fields of biology, medical science and geology all delved into "fractal search" in their respective fields. But the fervor eventually cooled off.

**■ Participation of Physicists: Early 1980s**

Physicists came to reassess the fractal concept by asking "why" and "in what manner" complicated phenomena occur.

**■ Back Again to Fractal: Latter 1980s**

Researchers belonging to the fields of biology, medical science and global physics start re-directing their attention on fractal from the perspective of "why" and "in what manner."

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Let's see in what procedures fractal research is conducted in connection with "why" and "in what manner."

"Firstly, complicated natural phenomena are scrutinized to draw out the mechanisms which are deemed to be essential. Next, the rules and formulas are established to permit expression of these mechanisms, and computers are used. These results are compared with the results of experiments which actually regenerate these natural phenomena, and studies are advanced to verify the propriety of these rules and formulas. For this, the three methods of natural observation, experiments and simulation with computers are applied."

However, why were the researchers of the latter 1970s incapable of continuing to retain their vision on "why" and "in what manner"?

"No, I feel they naturally did retain their vision. Only, in those days, there were no simple systems enabling actual verification. Even regarding fractal coastlines and blood networks, there was no way of engaging in various related experiments. In this regard, we physicists are simple-minded, so we first conceived a most simple system and strove to discover what lies in the background of complexity. While not really element reduction and also while different from current complexity systems, simple behaviors are searched for from complexity. Come to think of it, the clouds and mountain ranges which were originally conceived as being complicated were later learned to have a simple regularity. The basic attitude is none different even today."

## Closing In on the Fractal Question of "Why" Numerical Formulas Penetrate the Natural World

With the pair of photographs shown on page 7, the one at the left shows a bacteria colony, and the one at the right a computer-simulated image. It can be understood that they are closely-resembling fractal patterns. The forms of complicated patterns in the natural world can be described with numerical formulas (algorithms), and their secrets resolved by imitation.

"Persons who are weak in algorithms may well understand models as being something like the rule of the chess game. It is simple rules. There are no detailed parameters as in connection with how branching should be done when a bacteria colony grows by a certain extent, or what to do about the size. This is because our objective is not to simply get a resemblance but rather to probe for an essential mechanism. Therefore, as explained earlier, it is necessary not only to face a computer but to observe the natural world in details."

Another characteristic of algorithms is that they are abstract. For example, it is reported that a description in algorithms provides an intuition that a colony and a thunderbolt which appear substantially different from each other as observed from phenomenology may actually be following the same pattern-forming mechanism. But why is it that biological and non-biological phenomena resemble each other so closely?

"When describing the mechanism of growth of bacteria colonies, we perceive that ordinarily, the colony itself grows and determines the rule of branching. But our simulation is different. It is so set that branching occurs as a result of the aggregation of several peripheral conditions. Therefore, it may be that in the real world, the initiative of growth lies not with the growth side but rather with the surrounding environments such as light and air, in the same manner that a thunderbolt is branched in conformance with the state of the electric field."

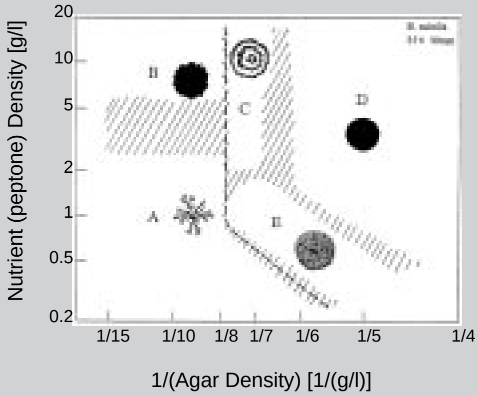
Numerical formulas penetrate the natural world, regardless of whether a biological or a non-biological phenomenon is involved. Engaging in such research permits us to approach the basic question of why nature prefers fractal modes. "It may be that we all desire to resolve questions, as long as we are scientists."

Professor Matsushita has also unexpectedly discovered a breakthrough in the process of his research on colonies.

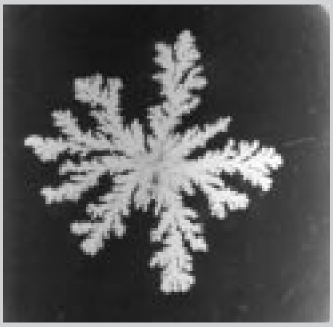
"The fact is that bacteria colonies assume a fractal pattern when the culturing bed gel is hard, or difficult to move about in, and when the nutrition level is low. Therefore, changing the environmental conditions and scrutinizing how patterns are crumbled may provide a lead to elucidating the essential mechanism when the colonies conversely assume fractal patterns. This is our ordinary method of approach when engaging in experiments. Under

**■Figure 3　Pattern of Bacteria Colony (hay bacillus) Changes Due to Environmental Changes**

The nutrition in gel increases the higher up, and the gel gets softer to the right to permit free motions. Environment A indicates a fractal pattern, and a different pattern is assumed in other environments.




**■ Colony pattern of bacteria cultured for a period of one month (hay bacillus).**



**■ Image simulated with computer**

this system, several patterns appear depending on the surrounding environment (see Figure. 3), or a regularity exists, separate from fractal pattern. This means that a bigger regularity exists in their background for selecting the regularity in specific environments. It appears that dubious matters are increasing or that the situation is becoming increasingly enthusiastic."

He suggests definite response in a bright tone, and the following conversation was exchanged when we departed from each other.

"There is a theory that the ambulation of an intoxicated person is also fractal. I'm conducting experiments about three time a week, but when observing that I'm reaching home unmistakably, I feel that the fractal pattern is not involved.

When dust is removed from a well-accustomed scenic photograph, we get a picture of brilliant color. The talks given by these professors display the same refined brilliancy. How would Newton have felt if he knew of this situation, who was obsessed with alchemy in order to acquire a new world view?